The Cylindrical Sundials of the Archaeological Museum of Athens

Evangelia Panou[1]
Efstratios Theodossiou[1]
Vassilios N. Manimanis[1]
Konstantinos Kalachanis[1]

Abstract

Sundials were the first instruments constructed by the ancient Greek astronomers for the measurement of time. The two ancient sundials described in this work are marble ones of the cylindrical type; they are currently kept in the sculpture storage room of the National Archaeological Museum of Athens (index catalog numbers 3159 and 3220). Their gnomons have been lost. Characteristic parameters of these sundials, such as the angle formed between the cone's axis and the generatrix, the geographical latitude of operation and the gnomon's length, are calculated from measurements of their geometrical dimensions.

Keywords: sundial, cylindrical, Athens, geographical latitude of operation

1. Ancient Sundials

The ancient Greeks constructed many exquisite timepieces, some of which exist even today. Based on the method used for the measurement of time, they belong to two general categories: sundials and hydraulic clocks.

The sundials measure time by using the apparent motion of the Sun on the celestial sphere and therefore their function depends directly on the position of the Sun in the sky. In Greek they are called *heliaka horologia* ("solar clocks") and also *skiathera* (means in Greek "chasing the shadow"); a term initially referred by the Pre-Socratic philosopher, Anaximander, who was the first to construct the sundial. The first sundial was a tall vertical column, called *gnomon* (Diogenes Laertius, *Vitae*, II, 1-7), and the subsequent sundials evolved from this primal design.

A classical sundial consists of a dial's surface and a gnomon placed upon it at an angle that depends on the geographical latitude of the place. Based on the dial's geometry and the angle it forms with the gnomon, sundials are divided into subtypes, such as:

1. hemispheric sundial with central gnomonic point
2. truncated spherical sundial with central gnomonic point
3. quarter-spherical sundial with central gnomonic point
4. spherical sundial
5. conical sundial and
6. cylindrical sundial.

[1] Department of Astrophysics-Astronomy and Mechanics, School of Physics, National and Kapodistrian University of Athens, Panepistimiopolis, Zographos 157 84, Athens, Greece

Additionally, there were sundials without a gnomon; in this case the path of the Sun was indicated on the dial by the solar rays that passed though a tiny hole. The position of this hole depended on the geometry of the dial's surface.

A sundial measured the daylight hours in its place, from some minutes after sunrise to some minutes before sunset from the position of the gnomon's shadow on the dial's surface. Moreover, it was possible to calculate the length of time spans during daylight hours by the shift of this shadow. However, the length of the day, i.e. the period of time during which the Sun was above the horizon of the place, was not constant throughout the year; so the sundial measured hours of unequal length.

The use of sundials in ancient Greece for the measurement of time was very common. Several eminent Greek astronomers invented and constructed not only sundials, but also a variety of astronomical instruments like dioptra and astrolabes. In ancient Greece the sexagesimal numbering system was prevalent and one solar day was divided into 12 daylight hours (whose length varied throughout the year), and 12 night hours of similarly varying length. The hour was of different time duration, depending on the season of the year.

Apart from showing the daylight hour, sundials offered the possibility to pinpoint the solstices and the equinoxes. This was possible with the use of a grid of lines called the *analemma* (Vitruvius Marcus Pollio, *De Architectura*, Book IX, Ch. 1 par. 1). The shape of the dial's surface, on which this grid of lines was carved, was a part of a sphere, a cone or of a plane; accordingly, the sundials were called spherical, conical and planar. Depending on the position of their gnomon, the sundials could be horizontal, vertical or with inclined gnomon; additionally, depending on their size and volume they were classified as portable and mounted sundials.

The great number and variety of the ancient sundials attracted the interest of subsequent researchers into compiling lists of Greek and Roman sundials. The best-known lists of such sundials are the one compiled by Vitruvius (1st century BC) and the one by archaeologist Sharon Gibbs (Gibbs, 1976). In the catalog by Gibbs 263 sundials are numbered and classified according to their characteristics; each sundial in this list is identified by its number in the list accompanied by a capital G.

2. Characteristics of the Sundials

The ancient sundials consisted of their mounting or base, their dial, the gnomon and, in some cases, also of an analemma. The choice of the mounting and the surface of the dial and the placing of the gnomon defined the name of the instrument. A sundial could be spherical, hemispheric, quarter-spherical sundial with central gnomonic point, spherical sundial, conical sundial, cylindrical, annular, disk-shaped, bar-shaped, vertical of various orientations, etc. The grid consisted usually of 11 straight lines, the hour lines. Having a common starting point, the hour lines spread at 15-degree spans, forming a semicircle. Each line corresponded to a daylight hour and the three curves that sometimes existed and intersected the lines corresponded (from the gnomon's hole outwards) to the winter solstice, the two equinoxes and the summer solstice. The centre of the three curves was the intersection point of the 11 hour lines, i.e. the hole of the gnomon. During the other days of the year the solar shadow moved across arcs between the curves.

3. Orientation of the Sundials

In order to function properly, a sundial should be mounted with the proper orientation towards the north or the south, according to its type. The orientation is achieved without any difficulty, since it suffices to determine the direction of the meridian of the place:

A vertical gnomon was placed upon a horizontal flat surface. At one moment before true noon a point of the gnomon's shadow was marked and a circle's arc centered on the gnomon's base was traced, which passed from that point. Then, they marked the point where the gnomon's shadow in the afternoon intersected this arc. The part of the arc between the two marks was bisected into two equal parts and then the line between its middle point and the gnomon's base was the meridian line of the place, from which the meridian plane passed. So the sundial was mounted in such a way that its line of noon was on the vertical plane that passed from the meridian line.

4. Cylindrical Sundials

The special characteristic of the cylindrical sundials is the cylindrical shape of their dial's surface, which is a part of the surface of a cylinder. The shadow is cast by a gnomon. Depending on the orientation of the cylinder's axis the cylindrical sundials belong to one of two subtypes: inclined, whose cylinder's axis is parallel to the axis of the Earth, and vertical, in which the cylinder's axis is perpendicular to the axis of the Earth.

The form of the line and curve network on the dial depends on the subtype. In the case of the inclined cylindrical sundials the hour lines are circular arcs that are parallel to the plane of the equator and the curves of the solstices are at equal distances from the curve of the equinoxes. In the case of the vertical cylindrical sundials both the shape of the hour lines and the positioning of the gnomon are complicated, unless the sundial is of southern orientation.

5. The ancient Greek cylindrical sundials of the National Archaeological Museum of Athens

In the National Archaeological Museum of Athens are kept the following two ancient Greek cylindrical sundials, with index catalog numbers 3220 and 3159.

5.1. The National Archaeological Museum of Athens sundial with index no. 3220

The ancient Greek sundial of the National Archaeological Museum of Athens with index catalog no. 3220 is of apparently cylindrical type.

It dates from the Roman or the late ancient period (Kraus, 1991) and the location of its discovery is unknown. It is made of white marble and it incorporates a marble base; its weight is 29.70 Kg.

The dial's height ranges between 22 and 23 cm. Its width is 43.0 cm, while its depth is 18 cm. The length of the back arc-like semiperimeter is 63.0 cm.

The sundial's base is 4.5 cm to 6.5 cm wide and 1.5 cm to 4.4 cm deep. Both of its sides bear lion's feet; the foot of the left-hand side is 10.3 cm high, 6.5 cm wide and 4.2 cm deep, while the foot of the right-hand side has dimensions 10.8 cm × 6.4 cm × 4.4 cm, respectively, with some minor surface alterations. The distance from the lower part of the base to the lowest point of the dial's surface is 7.0 cm.

The dial's surface is 3.0 cm thick and is not preserved in good condition. Nine hour lines are clearly visible on it, while the curves of the solstices and the equinoxes are not visible. On the upper part of the dial's surface there is an almost horizontal line from which stem the 9 hour lines.

A striking incision is also visible on the dial's surface, which starts from the middle of the 2nd hour line; it approaches the horizontal line and end just beyond the 6th hour line.

The sundial's gnomon has been lost. After the end of the 5th hour line there is a circular hole; the inner diameter of the hole is 2.8 cm, while its outer rim, which is heavily weathered, has a diameter of 4.2cm.

The lengths of the hour lines, the distances between them as measured on the dial's upper straight line and the lengths of the arcs formed on the (uncarved) curve that passes from the center of the hole, from the intersection of the hour lines with these curves, are given in Table I.

Table I: Lengths of the hour lines and of arcs on the curves of the sundial No. 3220

Number of hour line (left-to-right)	Length of hour line (cm)	Spaces between successive hour lines	Distances of hour lines measured on the upper line of the dial (cm)	Lengths of the arcs on the (uncarved) curve passing from the center of the hole (cm)
1st	–	Edge – 1st	–	–
2nd	5.6	1st – 2nd	4.3	–
3rd	13.0	2nd – 3rd	3.2	3.4
4th	13.4	3rd – 4th	2.6	4.3
5th	14.4	4th – 5th	2.5	4.0
6th	10.6	5th – 6th	2.4	4.0
7th	14.7	6th – 7th	2.5	4.6
8th	14.4	7th – 8th	2.5	4.0
9th	12.3	8th – 9th	2.5	3.7
10th	8.8	9th – 10th	3.5	4.4
11th	–	10th – edge	4.0	–

This sundial is not in good condition and its gnomon has been lost. As a result, it is impossible to calculate certain characteristic parameters of it, since even the curves of the solstices and the equinoxes are not visible. The dial itself resists its classification into some type of sundials; it just happens that its geometrical characteristics are closer to these of a cylindrical sundial. This sundial has been listed by Sharon Gibbs in her catalog under no. 3003G; she describes it as of "semi-cylindrical type" (Gibbs 1976, p. 223).

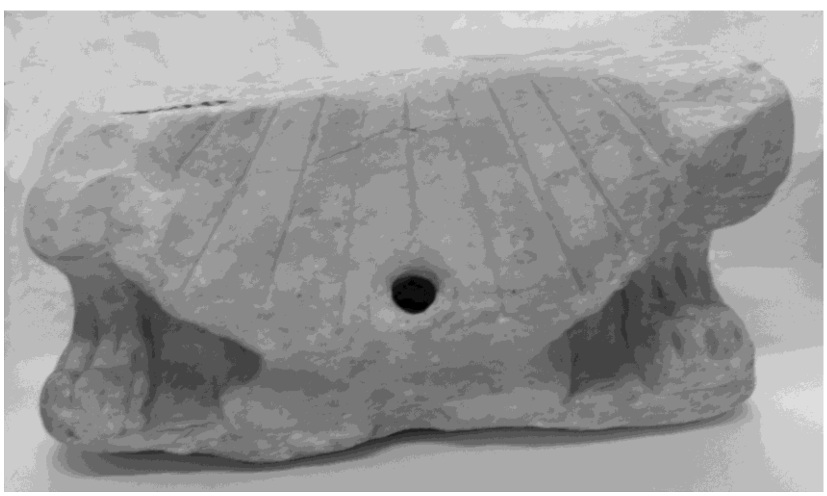

Figure 1: Sundial of apparently cylindrical type of the National Archaeological Museum of Athens (No. 3220).

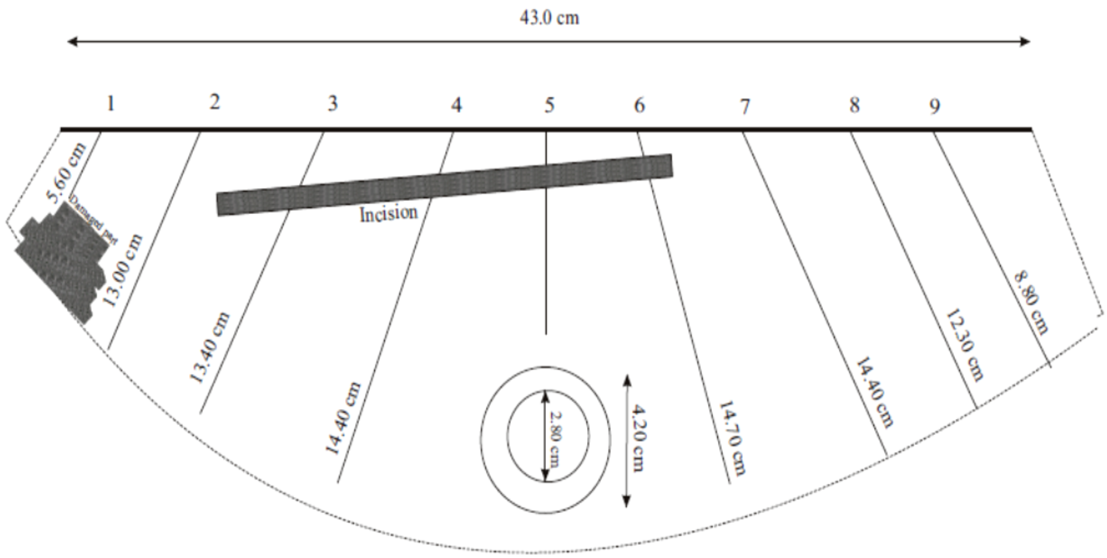

Figure 2: Drawing of the line grid of the apparently cylindrical sundial of the National Archaeological Museum of Athens (No. 3220).

5.2. The National Archaeological Museum of Athens sundial with index no. 3159

The ancient Greek sundial of the National Archaeological Museum of Athens with index catalog no. 3159 is of apparently cylindrical type.

It was discovered in the ancient Dionysus Theater, which is located under the southern side of the Acropolis of Athens and it is the oldest stone theater in the world. The dial is arguably dated in the late antiquity (Kraus, 1991). It is made of marble and it incorporates a marble base; its weight is 7.10 Kg.

The dimensions of height, depth and width are unequal. The height depth at the right-hand side of the construction is 13.7 cm, while at the left-hand side it is 15.6 cm. The depth at the right-hand side of the dial's surface is 12.0 cm, while at the left-hand side it is 9.5 cm. The width of the sundial's front side is 25.4 cm and that of the back side is 25.2 cm.

The dial is 2.2 cm thick, as measured along the direction of the meridian line. Eleven hour lines, the winter solstice curve and the curve of the equinoxes are clearly visible on it; the summer solstice curve does not exist.

The sundial's front side is weathered in places, especially on the right-hand side of the dial, where a part of the 10th and the 11th hour lines beyond the curve of the equinoxes is missing. The largest dimensions of this missing part, which is roughly a parallelogram, are approximately 4.0 cm (height) by 3.5 cm (width). The left-hand side of the dial's surface bears a prominent crack accompanied by missing pieces of marble. Other minor missing parts can be identified up to the 5th hour line beyond the curve of the equinoxes. Even more minor damages can be observed throughout the dial's surface.

The base of the sundial is a parallelogram with its upper surface following the form of the dial that bears the hour lines. The distance from the lower part of the base to the lowest point of the dial is 3.0 cm.

The position of the gnomon's hole is 2.4 cm away from the winter solstice's line as measured along the meridian line, 12.0 cm from the exterior edge of the dial's left-hand side and 13.5 cm away from the exterior edge of the dial's right-hand side.

The lengths of the hour lines (from left to right) from their intersection with the curve of winter solstice to the curve of the equinoxes and from the curve of the equinoxes to the outer curved end of the dial are given in Table II.

Table II: Lengths of the hour lines of the sundial No. 3159

Number of hour line (left-to-right)	Length (XI) of hour line from the winter solstice curve to the equinox curve (cm)	Length of hour line from the equinox curve to the outer curved surface of the dial (cm)	Length of hour line from the winter solstice curve to the outer curved surface of the dial (cm)
1st	5.6	1.2	6.8
2nd	5.4	1.9	7.3
3rd	5.2	2.4	7.6
4th	5.0	2.8	7.8
5th	5.0	2.8	7.8
6th*	5.0	3.1	8.1
7th	5.0	3.2	8.2
8th	5.2	3.4	8.6
9th	5.4	3.6	9.0
10th	5.1	3.2	8.3
11th	4.8	1.8	6.6

* The sixth hour line corresponds to the meridian hour line.

The lengths of the arcs formed successively on the equinox and winter solstice curves, and on the curve of the dial's outer surface from the intersection of the hour lines with these curves are listed in Table III, along with the values of the angles formed between successive hour lines according to the formula: where l_x and l_I the arc lengths on the solstice and equinox curves that correspond to the angle θ.

Table III: Arc lengths measured on the curves

Between successive hour lines	Lengths of arcs on the winter solstice curve l_x (cm)	Lengths of arcs on the equinoctial curve l_I (cm)	Arc lengths on the curve of the dial's outer surface l (cm)	Angle θ (degrees)
Edge – 1st	–	–	–	–
1st – 2nd	–	2.0	2.3	–
2nd – 3rd	1.3	2.1	2.5	8.49
3rd – 4th	1.1	2.0	2.3	9.92
4th – 5th	1.2	2.0	2.6	9.17
5th – 6th	1.2	2.2	2.9	11.46
6th – 7th	1.2	2.2	3.1	11.46
7th – 8th	1.2	2.1	3.0	10.31
8th – 9th	1.1	2.2	3.1	12.12
9th – 10th	1.1	1.9	2.4	8.49
10th – 11th	1.2	1.8	2.4	6.74

A characteristic distance upon the surface of the dial along the meridian line is the distance from the sundial's uppermost point to the curve of the winter solstice (2.4 cm).

This sundial (index no. 3159) is not in good condition and its gnomon has been lost. As a result, it is impossible to calculate the obliquity of the ecliptic ε. We cannot rely on the probability that it has characteristics corresponding to the other sundials of the National Archaeological Museum of Athens with index numbers 3156 (Panou et al., 2014), 3157 (Panou et al., 2014) and 3158 (Panou et al., 2013), although they were also retrieved from the Dionysus Theater under the Acropolis, because they, too, differ in their characteristics.

This sundial has not been listed by Sharon Gibbs in her catalog (Gibbs 1976, p. 223), as opposed with the other three sundials (3156, 3157 and 3158) found in the Theater's area.

The sundial no. 3159 has not a visible summer solstice curve. It is possible that this curve was on the missing part of the dial. If we posit that the summer solstice curve was very near the existing outer curve of the sundial, then the sundial does not conform to the characteristics of the conical sundials. However, it is very probable that it has the characteristics of an *inclined cylindrical* sundial; this is a subtype of the conical sundials with zero angle ($\omega = 0$) between the axis and the generatrix of the conical surface.

From the fact that the summer solstice curve could be very close to the outer curve of the dial, it follows that the curves of the winter and of the summer solstice is possible to be equidistant from the equinoxes curve along the meridian hour line The value for the obliquity of the ecliptic, the latitude φ and the gnomon's length d of the cylindrical sundial were computed from the formulae:

$$\phi = arc\tan[\cos(90^o \cdot \frac{l_x}{l_I}) \cdot \cot \varepsilon]$$

where: $$d = \sqrt{(\Gamma \mathrm{I})^2 + (OI)^2}$$

ε: obliquity of the ecliptic

OI: the distance (along the meridian hour line) of the gnomon's apex from the curve of the equinoxes, which is equal to:

$$\mathrm{OI} = \frac{12 \cdot l_I}{\pi}$$

ΓI: the distance (along the meridian hour line) from the gnomon's hole to the curve of the equinoxes

OXΓ: the external angle of the triangle OIX

l_I: the length of the arc along the curve of the equinoxes

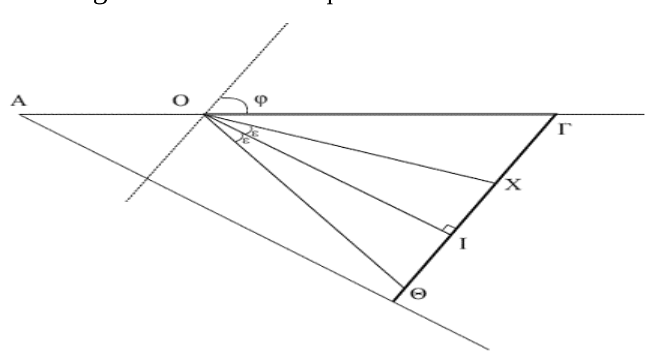

Figure 3: The dial's surface of a cylindrical sundial.

In Table IV are given the values obtained for the characteristic parameters of the sundial by applying the above formulae.

Table IV: Values of the characteristic quantities *ε*, *d* and *φ*	
Obliquity of the ecliptic *ε*	30.75°
Geographical latitude*φ*	47.75°
Length of gnomon *d*	11.2 cm

Figure 4: The apparently cylindrical sundial of the National Archaeological Museum of Athens with index no. 3159.

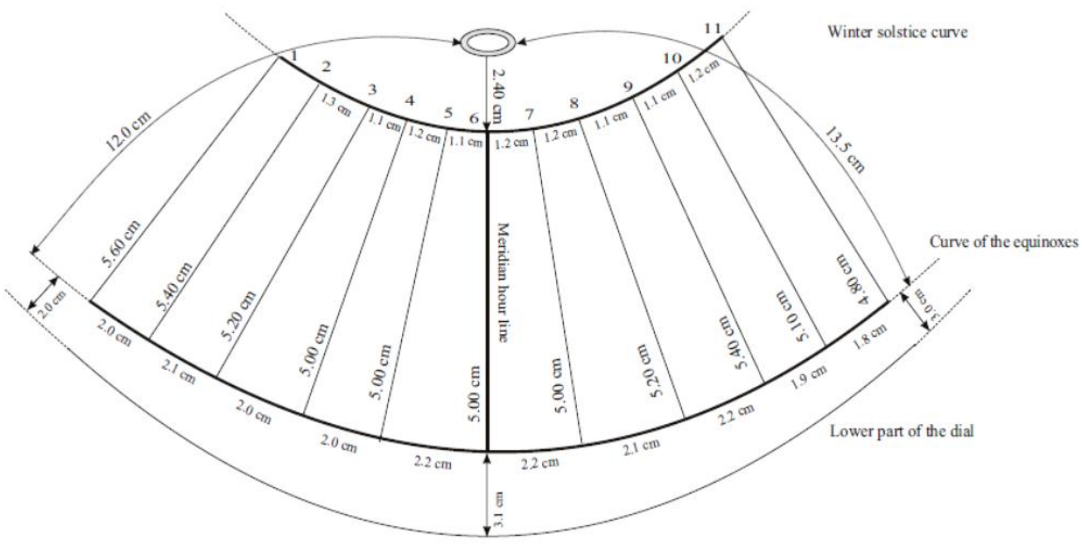

Figure 5: Drawing of the line grid of the cylindrical sundial of the National Archaeological Museum of Athens with index no. 3159.

6. Conclusions

The sundial with index no. 3220 is not in good condition and its gnomon has been lost. From the geometry of the visible curves of its dial's surface this sundial can be classified as of cylindrical sundial.

The sundial with index no. 3159 is in a much better state, although its gnomon has also been lost. A special question rises about its summer solstice curve, because it is not visible in the dial's surface; it is very probable that the curve was carved on the missing part of this surface. Under the assumption that the summer solstice curve was so close to the dial's external curve, the sundial can be classified as of cylindrical sundial. According to our calculations it could be used at geographical latitudes of *circa* 47^o. There is no region with such latitudes in the wider ancient Greek world, including the region of southern Italy and Sicily (*Magna Graecia*); therefore it seems more logical to assume that the dial's part where the summer solstice curve was carved is missing. If the missing part is large, then the sundial can be also classified as a conical one.

Acknowledgments

We thank very much sculptor Mr. Athanassios Kalantzis, art conservator Mr. Michael Demetreadis, archaeologist Mrs. Hrysanthi Tsoulis and the supervisor of the Prehistoric and Classical Antiquities of the National Archaeological Museum of Athens Mrs. Helen Kourinos for their help during the collecting of data.

References

Diogenes Laërtius (1964). Vitae Philosophorum, 2 vols: Oxford: Clarendon Press.

Gibbs Sh., (1976). Greek and Roman Sundials, Yale University Press, New Haven and London.

Kraus Th., (1991). Katalog der Einzelfunde. Series Karthago I, pp. 97-98.

Panou, E., Theodossiou, E., Manimanis, N., V., & Mantarakis, Z., P., (2013). The conical sundial of the Archaeological Museum of Athens. North American Sundial Society bulletin (NASS), Vol. 20 No 4, pp. 30-37.

Panou, E., Theodossiou, E., Manimanis N. V., & Kalachanis, K., (2014). Two conical sundials with missing gnomons in the Archaeological Museum of Athens. British Sundial Society (BSS) Bulletin Volume 26(i), March issue.

Vitruvius Marcus Pollio, (1969). De Architectura (translation in French and commentary by Jean Soubiran). Paris.